





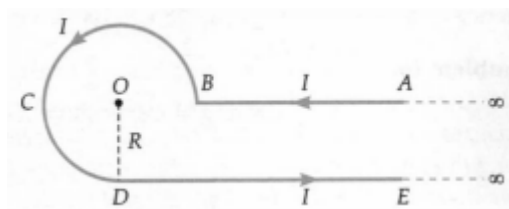


$10^{-14}$  m. Calculate the energy of the  $\alpha$ -particle.

21. An electric current is flowing due south along a power line. What is the direction of the magnetic field at a point [2]  
(a) above it and (b) below it?

OR

A current  $I$  is flowing in an infinitely long conductor bent into the shape shown in Fig. If the radius of the curved part is  $R$ , find the magnetic field at the centre  $O$ .



### Section C

22. What is Wheatstone bridge? Deduce the condition for which Wheatstone bridge is balanced. [3]  
23. Write any two distinguishing features between conductors, semiconductors and insulators on the basis of energy band diagrams. [3]  
24. Red light, however bright it is, cannot produce the emission of electrons from a clean zinc surface. But even weak ultraviolet radiation can do so. Why? [3]

Electrons are emitted from the cathode of negligible work function, when photons of wavelength  $\lambda$  are incident on it. Derive the expression for the de Broglie wavelength of the electrons emitted in terms of the wavelength of the incident light.

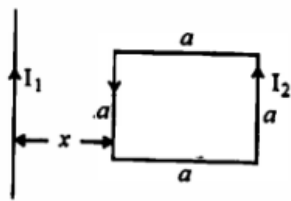
25. Draw a plot of potential energy of a pair of nucleons as a function of their separations. Mark the regions where the nuclear force is [3]  
i. attractive and  
ii. repulsive.

Write any two characteristic features of nuclear forces.

26. Show that the radius of the orbit in hydrogen atom varies as  $n^2$ , where  $n$  is the principal quantum number of the atom. [3]  
27. Monochromatic light of wavelength 600 nm is incident from air on a glass surface. What are the wavelength, frequency and speed of refracted light? Refractive index of glass 1.5. [3]  
28. A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of  $8 \text{ cm s}^{-1}$  in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of  $10^{-3} \text{ T cm}^{-1}$  along the negative x-direction (that is it increases by  $10^{-3} \text{ T cm}^{-1}$  as one move in the negative x-direction), and it is decreasing in time at the rate of  $10^{-3} \text{ T s}^{-1}$ . Determine the direction and magnitude of the induced current in the loop if its resistance is  $4.50 \text{ m}\Omega$ . [3]

OR

- i. Define mutual inductance and write its S.I. unit.  
ii. A square loop of side 'a' carrying a current  $I_2$  is kept at distance  $x$  from an infinitely long straight wire carrying a current  $I_1$  as shown in the figure. Obtain the expression for the resultant force acting on the loop.



### Section D

29. Read the text carefully and answer the questions:

[4]

Maxwell showed that the speed of an electromagnetic wave depends on the permeability and permittivity of the medium through which it travels. The speed of an electromagnetic wave in free space is given by  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ . The fact led Maxwell to predict that light is an electromagnetic wave. The emergence of the speed of light from purely electromagnetic considerations is the crowning achievement of Maxwell's electromagnetic theory. The speed of an electromagnetic wave in any medium of permeability  $\mu$  and permittivity  $\epsilon$  will be  $\frac{c}{\sqrt{K\mu_r}}$  where  $K$  is the dielectric constant of the medium and  $\mu_r$  is the relative permeability.

(i) The dimensions of  $\frac{1}{2}\epsilon_0 E^2$  ( $\epsilon_0$  : permittivity of free space;  $E$  = electric field) is

- |                 |                    |
|-----------------|--------------------|
| a) $MLT^{-1}$   | b) $ML^{-1}T^{-2}$ |
| c) $ML^2T^{-2}$ | d) $ML^2T^{-1}$    |

(ii) Let  $[\epsilon_0]$  denote the dimensional formula of the permittivity of the vacuum. If  $M$  = mass,  $L$  = length,  $T$  = time and  $A$  = electric current, then

- |  |                                      |
|--|--------------------------------------|
| a) $[\epsilon_0] = ML^2T^{-1}$         | b) $[\epsilon_0] = MLT^{-2}A^{-2}$   |
| c) $[\epsilon_0] = M^{-1}L^{-3}T^4A^2$ | d) $[\epsilon_0] = M^{-1}L^{-3}T^2A$ |

(iii) An electromagnetic wave of frequency 3 MHz passes from vacuum into a dielectric medium with permittivity  $\epsilon = 4$ . Then

- |  |   |
|--|---|
| a) wavelength is halved and the frequency remains unchanged. | b) wavelength and frequency both remain unchanged       |
| c) wavelength is doubled and the frequency remains unchanged | d) wavelength is doubled and the frequency becomes half |

OR

Which of the following are not electromagnetic waves?

cosmic rays,  $\gamma$ -rays,  $\beta$ -rays, X-rays

- |                   |                |
|-------------------|----------------|
| a) $\beta$ -rays  | b) X-rays      |
| c) $\gamma$ -rays | d) cosmic rays |

(iv) The electromagnetic waves travel with

- |   |  |
|---|--|
| a) the speed of light $c = 3 \times 10^8 \text{ m s}^{-1}$ in fluid medium. | b) the speed of light $c = 3 \times 10 \text{ m s}^{-1}$ in solid medium |
| c) the speed of light $c = 3 \times 10^8 \text{ m s}^{-1}$ in free space    | d) the same speed in all media   |

30. Read the text carefully and answer the questions:

[4]

Gauss's law and Coulomb's law, although expressed in different forms, are equivalent ways of describing the relation between charge and electric field in static conditions. Gauss's law is  $\epsilon_0 \phi = q_{\text{end}}$ , when  $q_{\text{end}}$  is the net charge inside an imaginary closed surface called Gaussian surface.  $\phi = \oint \vec{E} \cdot d\vec{A}$  gives the electric flux through the Gaussian surface. The two equations hold only when the net charge is in vacuum or air.



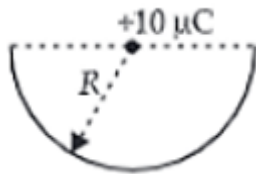
(i) If there is only one type of charge in the universe, then ( $\vec{E} \rightarrow$  Electric field,  $d\vec{s} \rightarrow$  Area vector)

- |  |  |
|--|--|
| a) $\oint \vec{E} \cdot d\vec{s} \neq 0$ on any surface  | b) $\oint \vec{E} \cdot d\vec{s}$ could not be defined         |
| c) $\oint \vec{E} \cdot d\vec{s} = 0$ if charge is outside,<br>$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$ if charge is inside | d) $\oint \vec{E} \cdot d\vec{s} = \infty$ if charge is inside |

(ii) What is the nature of Gaussian surface involved in Gauss law of electrostatic?

- |             |               |
|-------------|---------------|
| a) Magnetic | b) Scalar     |
| c) Vector   | d) Electrical |

(iii) A charge  $10 \mu\text{C}$  is placed at the centre of a hemisphere of radius  $R = 10 \text{ cm}$  as shown. The electric flux through the hemisphere (in MKS units) is



- |                     |                     |
|---------------------|---------------------|
| a) $20 \times 10^5$ | b) $10 \times 10^5$ |
| c) $6 \times 10^5$  | d) $2 \times 10^5$  |

(iv) The electric flux through a closed surface area  $S$  enclosing charge  $Q$  is  $\phi$ . If the surface area is doubled, then the flux is

- |                     |            |
|---------------------|------------|
| a) $\frac{\phi}{4}$ | b) $\phi$  |
| c) $\frac{\phi}{2}$ | d) $2\phi$ |

**OR**

A Gaussian surface encloses a dipole. The electric flux through this surface is

- |                           |                            |
|---------------------------|----------------------------|
| a) $\frac{q}{\epsilon_0}$ | b) $\frac{q}{2\epsilon_0}$ |
| c) zero                   | d) $\frac{2q}{\epsilon_0}$ |

### Section E

31. A thin equiconvex lens (radius of curvature of either face being 33 cm) is placed on a horizontal plane mirror and a pin held 20 cm vertically above the lens coincides in position with its own image. The space between the [5]

lower surface of the lens and the mirror is filled with a liquid and then, to coincide with the image as before, the pin has to be raised to a distance of 25 cm from the lens. Find the refractive index of the liquid.

OR

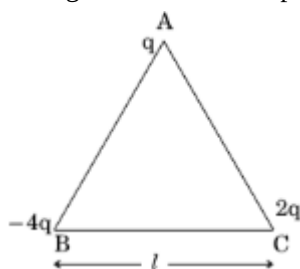
- i. In Young's double-slit experiment, describe briefly how bright and dark fringes are obtained on the screen kept in front of a double slit. Hence obtain the expression for the fringe width.
- ii. The ratio of the intensities at minima to the maxima in Young's double-slit experiment is 9:25. Find the ratio of the width of the slits.

32. Two tiny spheres carrying charges  $1.5\mu C$  and  $2.5\mu C$  are located 30 cm apart. Find the potential and electric field: [5]

- a. at the mid point of the line joining the two charges, and
- b. at a point 10 cm from this mid point in a plane normal to the line and passing through the mid point.

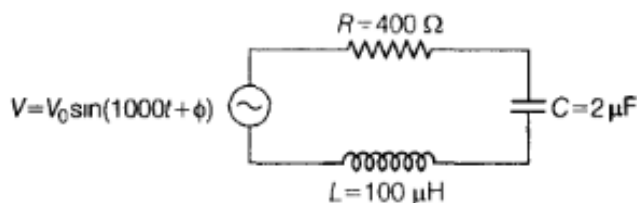
OR

- i. Three point charges  $q$ ,  $-4q$  and  $2q$  are placed at the vertices of an equilateral triangle ABC of side  $l$  as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge  $q$ .



- ii. Find out the amount of the work done to separate the charges at infinite distance.

33. i. Determine the value of phase difference between the current and the voltage in the given series L-C-R circuit. [5]



- ii. Calculate the value of additional capacitor which may be joined suitably to the capacitor C that would make the power factor of the circuit unity.

OR

- i. An alternating voltage  $V = V_m \sin \omega t$  applied to a series L-C-R circuit derives a current given by  $I = I_m \sin(\omega t + \phi)$ . Deduce an expression for the average power dissipated over a cycle.
- ii. For circuit used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

# Solution

## Section A

1. (a) three or less than three

**Explanation:** The electron theory states that all matter is composed of atoms and the atoms are composed of smaller particles called protons, electrons, and neutrons. The electrons orbit the nucleus which contains the protons and neutrons. It is the valence electrons that we are most concerned with in electricity. These are the electrons which are easiest to break loose from their parent atom. Normally, conductors have three or less valence electrons; insulators have five or more valence electrons; and semiconductors usually have four valence electrons.

- 2.

(b)  $10 \Omega$

**Explanation:** Connecting a resistance in series with the galvanometer does not affect the balanced condition of the bridge. We just have  $(10 \Omega + 10 \Omega)$  and  $(10 \Omega + 10 \Omega)$  resistances in parallel.

$$\therefore R_{eq} = \frac{20 \times 20}{20 + 20} = 10 \Omega$$

- 3.

(c) high resolving power

**Explanation:** Resolving power is directly proportional to aperture.

- 4.

(b)  $9.27 \times 10^{-24} \text{ Am}^2$

**Explanation:** 1 Bohr magneton

$$\begin{aligned} &= \frac{eh}{4\pi m_e} \\ &= \frac{1.6 \times 10^{-19} \times 6.62 \times 10^{-34}}{4\pi \times 9.1 \times 10^{-31}} \\ &= 9.27 \times 10^{-24} \text{ Am}^2 \end{aligned}$$

- 5.

(d) there is always loss of energy

**Explanation:** Let the two conductors have capacitances  $C_1$  and  $C_2$  and let their potentials be  $V_1$  and  $V_2$ .

Total initial energy of the conductors  $U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$ .

When two charged conductors are connected by a wire, the flow of charge stops when the potentials are equalized.

Let  $V$  be the common potential and  $Q_1$  and  $Q_2$  be the charges on the conductors after the common potential is attained.

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

The final energy

$$U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(C_1 + C_2) \left( \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \right)^2 = \frac{1}{2} \frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$U_f - U_i = \frac{1}{2} \left[ \frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2} - (C_1V_1^2 + C_2V_2^2) \right] = -\frac{1}{2} \frac{C_1C_2(V_1 - V_2)^2}{C_1 + C_2}$$

- 6.

(c) 8 N in -z direction.

**Explanation:**  $\vec{F} = q(\vec{v} \times \vec{B})$

$$= -2 \times 10^{-6} \text{ C} \left[ (2\hat{i} + 3\hat{j}) \times 10^6 \text{ ms}^{-1} \times 2\hat{j} \text{ T} \right]$$

$$= -8\hat{k}$$

i.e., 8 N in the -ve z direction.

7. (a) 10 V

**Explanation:** As induced emf,  $|e| = \frac{d\phi}{dt}$

$$= \frac{d}{dt}(5t^2 + 3t + 16)$$

$$= 10t + 3$$

So, at  $t = 3\text{ s}$ , induced  $|e|$  is  $= 10 \times 3 + 3 = 33\text{ V}$



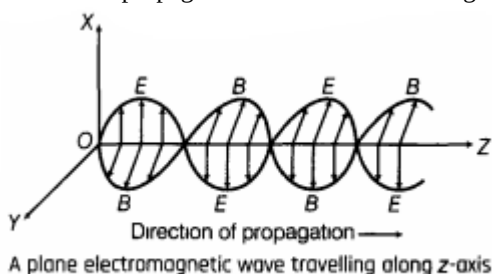
So, at  $t = 4\text{s}$ , induced  $|e|$  is  $= 10 \times 4 + 3 = 43\text{V}$

Therefore emf induced in the fourth second is given by  $= 43 - 33 = 10\text{V}$

8. (c)  $1.28\text{ Am}^2$   
**Explanation:**  $m = NIA$   
 $= 2000 \times 1.6 \times 10^{-4} \times 4$   
 $= 1.28\text{ Am}^2$
9. (d) by using white light instead of monochromatic light  
**Explanation:** In a young's double-slit experiment the central bright fringe is identified by using white light instead of monochromatic light.
10. (c)  $-q$   
**Explanation:** Force on  $q$  due to  $4q$ ,  
 $F_1 = \frac{1}{4\pi\epsilon_0} \frac{4q^2}{d^2}$   
Force on  $q$  due to  $Q$ ,  
 $F_2 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{d^2/4} = \frac{1}{4\pi\epsilon_0} \frac{4Qq}{d^2}$   
For equilibrium,  $F_1 + F_2 = 0$  (resultant force is 0)  
 $\frac{1}{4\pi\epsilon_0} \frac{4Qq}{d^2} + \frac{1}{4\pi\epsilon_0} \frac{4q^2}{d^2} = 0$   
Hence on solving we get,  $Q = -q$
11. (a) 10 volt  
**Explanation:** The forward biased p-n junction does not offer any resistance.  
 $\therefore R_{AB} = \frac{10 \times 10}{10 + 10} = 5\text{ k}\Omega$   
Total resistance,  
 $R = 10 + 5 = 15\text{ k}\Omega$   
Current in the circuit,  
 $I = \frac{V}{R} = \frac{30\text{ V}}{15 \times 10^3} \text{ A} = 2 \times 10^{-3} \text{ A}$   
Current through each arm  $= \frac{I}{2} = 10^{-3} \text{ A}$   
 $\therefore V_{AB} = 10 \times 10^3 \times 10^{-3} = 10\text{ V}$
12. (d)  $60^\circ$   
**Explanation:**  $60^\circ$
13. (a) Both A and R are true and R is the correct explanation of A.  
**Explanation:** Electrons being emitted as photoelectrons have different velocities. Actually, all the electrons do not occupy the same level of energy but they occupy continuous band and levels. So, electrons being knocked off from different levels come out with different energies. The work function is the energy required to pull the electron out of the metal surface. Naturally, electrons on the surface will require less energy to be pulled out hence will have lesser work function as compared with those deep inside the metal.  
So, Both A and R are true and R is the correct explanation of A.
14. (c) A is true but R is false.  
**Explanation:** Potential and potential energy are different quantities and cannot be equated.
15. (b) Both A and R are true but R is not the correct explanation of A.  
**Explanation:** Both A and R are true but R is not the correct explanation of A.
16. (a) Both A and R are true and R is the correct explanation of A.  
**Explanation:** Both A and R are true and R is the correct explanation of A.

#### Section B

17. In general, a moving charge can produce both electric and magnetic fields. So, an oscillating charge can produce oscillating electric and magnetic fields with the same frequency of the oscillating charge. Thus an electromagnetic wave is produced by an oscillating charge directed along the perpendicular to both the electric and magnetic field (two fields are also perpendicular to each other) and the direction of propagation is found from the cross product  $\vec{E} \times \vec{B}$ .  
In the diagram below, electric and magnetic fields vectors are vibrating along positive X and Y axes respectively with the direction of propagation of the em wave along the positive Z-axis.



18. i. The angle of dip decreases from  $90^\circ$  to  $0^\circ$  as we move from pole to the equator of earth.  
ii. For paramagnetic materials, magnetic lines of force are feebly attracted by the materials and for that reason only a few magnetic field lines enter in it. So, specimen X is paramagnetic. For ferromagnetic materials, all magnetic lines of force prefer to go through it. So, specimen Y is ferromagnetic.
19. **Advantages of semiconductor diodes over vacuum diodes:**
- No cathode heating is required in a junction diode for the production of charge carriers.
  - A p-n junction diode is much smaller in size, robust and compact, and hence easy to handle.
  - It starts operating as soon as it is switched on.
  - It can be used for much higher frequencies.
  - It dissipates less heat and is highly efficient in operation.
  - It has a much longer life.
  - The voltage drop across a junction diode is very small compared to that across a vacuum diode.
  - No vacuum has to be created for its operation.

**Disadvantage:** It is highly sensitive to temperature and gets burnt up even if a moderately high current is passed through it.

$$20. K = \frac{2kZe^2}{5_0} = \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{3.95 \times 10^{-14} \times 1.6 \times 10^{-13}} \text{MeV} = 6 \text{ MeV}$$

21. According to right hand rule, the direction of the field is (a) towards west above the wire and (b) towards east below the wire.

OR

As the point O lies on the straight part AB, So

$$B_{AB} = 0$$

$$B_{BCD} = \frac{\mu_0 I}{4\pi R} \cdot \frac{3\pi}{2}, \text{ acting normally outward}$$

$$B_{DE} = \frac{\mu_0 I}{4\pi R} (\sin 90^\circ + \sin 0^\circ) = \frac{\mu_0 I}{4\pi R}, \text{ acting normally outward}$$

Total magnetic field at the centre O

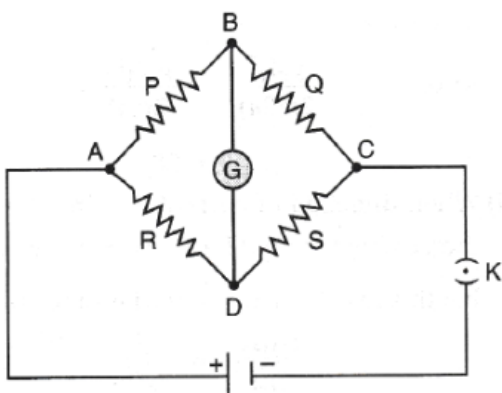
$$B = B_{AB} + B_{BCD} + B_{DE}$$

$$= 0 + \frac{\mu_0 I}{4\pi R} \cdot \frac{3\pi}{2} + \frac{\mu_0 I}{4\pi R}$$

$$\text{or } B = \frac{\mu_0 I}{4\pi R} \left( \frac{3\pi}{2} + 1 \right), \text{ acting normally outward.}$$

### Section C

22. The Wheatstone bridge is an arrangement of four resistances. In this bridge, four resistances are connected on four arms of a quadrilateral. Across one diagonal, a battery and key are connected. Across the second diagonal, a galvanometer is connected as shown in the figure. Consider P, Q, R, and S are four resistances connected on the sides AB, BC, AD, and DC of the quadrilateral respectively.



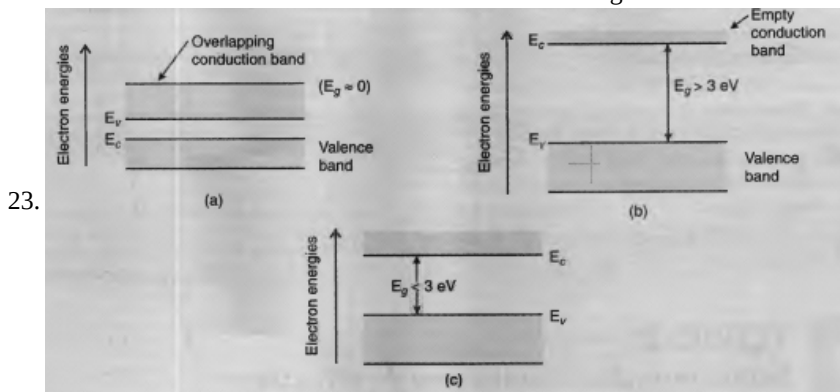
Galvanometer G is connected between points B and D and battery B is connected between A and C. Now in the balance condition, when the deflection in the galvanometer is zero in the closed mesh ABDA, then by applying Kirchhoff's law,  $I_1P - I_2P = 0$  or  $I_1P = I_2R$  ..... (i)

In closed mesh CBDC,

$$I_1Q = I_2S \text{ ..... (ii)}$$

Dividing (i) by (ii),  $\frac{P}{Q} = \frac{R}{S}$

This is the balanced condition for the Wheatstone bridge.



23.

Two distinguishing features are given below :

- i. In conductors, the valence band and conduction band tends to overlap (or nearly overlap) each other , while in insulators they are separated by a large energy gap and in semiconductors they are separated by a small energy gap.
- ii. The conduction band of a conductor, has a large number of electrons available for electrical conduction. However, the conduction band of insulators is almost empty while that of the semi-conductor has only a (very) small number of such electrons available for electrical conduction

24. The frequency of ultraviolet radiations is more while that of red light is less than the threshold frequency for a zinc surface, so ultraviolet radiations can cause the emission of electrons and red light cannot.

From Einstein's photoelectric equation, K.E. of a photoelectron is

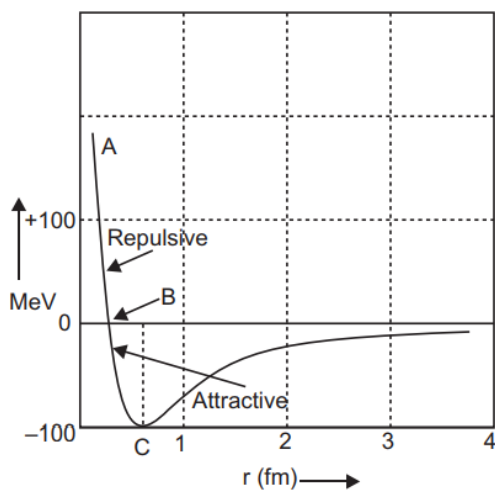
$$\frac{1}{2}mv^2 = h\nu - W_0 = h\nu - 0 = \frac{hc}{\lambda}$$

$$\text{or } v = \sqrt{\frac{2hc}{m\lambda}}$$

de Broglie wavelength of electrons,

$$\lambda_e = \frac{h}{mv} = \frac{h}{m \sqrt{\frac{2hc}{m\lambda}}} = \sqrt{\frac{h\lambda}{2mc}}$$

25. The following graph shows the variation of potential energy with the separation of nucleons



1. Part BC of the graph shows the attractive force.
2. Part AB of the graph shows the repulsive force.

The characteristic features of the nuclear force are as under:

1. Nuclear forces are attractive and stronger than the electrostatic force.
2. Nuclear forces are charge-independent and short range forces.

26. According to the Bohr's theory of hydrogen atom, the angular momentum of revolving electron is given by

$$mvr = \frac{nh}{2\pi} \dots\dots(i)$$

where, m = mass of the electron, v = velocity of the electron.

r = radius of the orbit, h = Planck's constant and n = principal quantum number of the atom.

If an electron of mass m and velocity v is moving in a circular orbit of radius r, then the centripetal force is given by

$$F_c = mv^2/r \dots\dots(ii)$$

Also, if the charge on the nucleus is Ze, then the force of electrostatic attraction between the nucleus and the electron will provide the necessary centripetal force

$$\Rightarrow F_c = F_e$$

$$\Rightarrow \frac{mv^2}{r} = \frac{ke^2}{r^2} \quad [ \because Z = 1 ]$$

$$\Rightarrow r = \frac{e^2 \cdot k}{mv^2} \dots\dots(iii)$$

$$r = \frac{ke^2 4\pi^2 m^2 r^2}{m \cdot n^2 h^2} \quad [ \text{from eq. (i)} ]$$

$$\Rightarrow r = \frac{n^2 h^2}{ke^2 4\pi^2 m} \Rightarrow r \propto n^2$$

27. During a refraction, the frequency remains unchanged. Both wavelength and speed get changed.

Frequency,

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ ms}^{-1}}{600 \times 10^{-9} \text{ m}}$$

$$= 5 \times 10^{14} \text{ Hz}$$

Speed of refracted light,

$$v_{\text{glass}} = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ ms}^{-1}$$

Wavelength of refracted light,

$$\lambda_{\text{glass}} = \frac{v_{\text{glass}}}{\nu}$$

$$= \frac{2 \times 10^8}{5 \times 10^{14}} = 4 \times 10^{-7} \text{ m} = 400 \text{ nm.}$$

28. The magnetic field in loop varies position 'x' of loop and also with time simultaneously. Side of the square loop, s = 12 cm = 0.12 m

$$\text{Area of the square loop, } A = 0.12 \times 0.12 = 0.0144 \text{ m}^2$$

$$\text{Velocity of the loop, } v = 8 \text{ cm/s} = 0.08 \text{ m/s}$$

Gradient of the magnetic field along negative x-direction,

$$\frac{dB}{dx} = 10^{-3} \text{ Tcm}^{-1} = 10^{-1} \text{ Tm}^{-1}$$

And, rate of decrease of the magnetic field,

$$\frac{dB}{dt} = 10^{-3} \text{ Ts}^{-1}$$

Resistance of the loop,  $R = 4.5 \text{ m}\Omega = 4.5 \times 10^{-3} \Omega$

The induced e.m.f produced due to magnetic field with time T:  $e_1 = \frac{d\phi}{dt} = A \frac{dB}{dt}$

$$e_1 = 1.44 \times 10^{-5} \text{ V}$$

Rate of change of the magnetic flux with distance X:  $e_2 = \frac{d\phi}{dt} = A \times \frac{dB}{dx} \times v$

$$= 1152 \times 10^{-4} \times 10^{-3} = 11.52 \times 10^{-5} \text{ V}$$

Since the rate of change of the flux is the induced emf, the total induced in the loop can be calculated as:

$$e = 1.44 \times 10^{-5} + 11.52 \times 10^{-5}$$

$$= 12.96 \times 10^{-5} \text{ V}$$

$\therefore$  Induced current,  $i = \frac{e}{R}$

$$= \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}}$$

$$i = 2.88 \times 10^{-2} \text{ A}$$

Hence, the direction of the induced is such that there is an increase in the flux through the loop along the positive z-direction.

OR

i. **Mutual inductance:** It is the property of the coils that enables it to oppose the changes in the current in another coil. Its S.I unit is Henry (H).

ii. Force on the part of the loop which is parallel to infinite straight wire and at a distance x from it.

$$F_1 = \frac{\mu_0}{2\pi} \frac{I_1 I_2 a}{x} \text{ (away from the infinite straight wire)}$$

Force on the part of the loop which is at a distance (x + a) from it

$$F_2 = \frac{\mu_0}{2\pi} \frac{I_1 I_2 a}{(x+a)} \text{ (towards the infinite straight wire)}$$

Net force  $F = F_1 - F_2$

$$F = \frac{\mu_0 I_1 I_2 a}{2\pi} \left[ \frac{1}{x} - \frac{1}{x+a} \right]$$

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2 a^2}{x(x+a)}$$

### Section D

#### 29. Read the text carefully and answer the questions:

Maxwell showed that the speed of an electromagnetic wave depends on the permeability and permittivity of the medium through which it travels. The speed of an electromagnetic wave in free space is given by  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ . The fact led Maxwell to predict that light is an electromagnetic wave. The emergence of the speed of light from purely electromagnetic considerations is the crowning achievement of Maxwell's electromagnetic theory. The speed of an electromagnetic wave in any medium of permeability  $\mu$  and permittivity  $\epsilon$  will be  $\frac{c}{\sqrt{K\mu_r}}$  where K is the dielectric constant of the medium and  $\mu_r$  is the relative permeability.

(i) **(b)**  $\text{ML}^{-1}\text{T}^{-2}$

**Explanation:**  $\frac{1}{2} \epsilon_0 E^2 = \text{energy density} = \frac{\text{Energy}}{\text{Volume}}$

$$\therefore \left[ \frac{1}{2} \epsilon_0 E^2 \right] = \frac{\text{ML}^2 \text{T}^{-2}}{\text{L}^3} = [\text{ML}^{-1}\text{T}^{-2}]$$

(ii) **(c)**  $[\epsilon_0] = \text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2$

**Explanation:** As  $\epsilon_0 = \frac{q_1 q_2}{4\pi F R^2}$  (from Coulomb's law)

$$\epsilon_0 = \frac{\text{C}^2}{\text{Nm}^2} \frac{[\text{AT}]^2}{\text{MLT}^{-2} \text{L}^2} = \text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2$$

(iii) **(a)** wavelength is halved and the frequency remains unchanged.

**Explanation:** The frequency of the electromagnetic wave remains same when it passes from one medium to another.

$$\text{Refractive index of the medium, } n = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{\frac{4}{1}} = 2$$

Wavelength of the electromagnetic wave in the medium,

$$\lambda_{\text{med}} = \frac{\lambda}{n} = \frac{\lambda}{2}$$

OR

**(a)**  $\beta$ -rays

**Explanation:**  $\beta$ -rays consists of electrons which are not electromagnetic in nature.

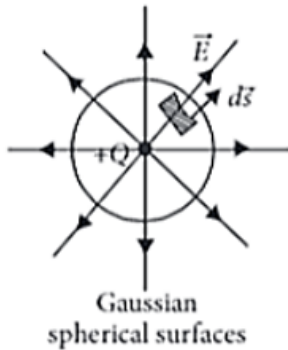
(iv) **(c)** the speed of light  $c = 3 \times 10^8 \text{ m s}^{-1}$  in free space

**Explanation:** The velocity of electromagnetic waves in free space (vacuum) is equal to velocity of light in vacuum

(i.e.,  $3 \times 10^8 \text{ m s}^{-1}$ ).

30. Read the text carefully and answer the questions:

Gauss's law and Coulomb's law, although expressed in different forms, are equivalent ways of describing the relation between charge and electric field in static conditions. Gauss's law is  $\epsilon_0 \phi = q_{\text{enc}}$ , when  $q_{\text{enc}}$  is the net charge inside an imaginary closed surface called Gaussian surface.  $\phi = \oint \vec{E} \cdot d\vec{A}$  gives the electric flux through the Gaussian surface. The two equations hold only when the net charge is in vacuum or air.



- (i) (c)  $\oint \vec{E} \cdot d\vec{s} = 0$  if charge is outside,  $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$  if charge is inside

**Explanation:** If there is only one type of charge in the universe then it will produce electric field somehow. Hence Gauss's law is valid.

- (ii) (c) Vector

**Explanation:** Vector

- (iii) (c)  $6 \times 10^5$

**Explanation:** According to Gauss's theorem,

Electric flux through the sphere =  $\frac{q}{\epsilon_0}$

$\therefore$  Electric flux through the hemisphere =  $\frac{1}{2} \frac{q}{\epsilon_0}$

$$= \frac{10 \times 10^{-6}}{2 \times 8.854 \times 10^{-12}} = 0.56 \times 10^6 \text{ N m}^2 \text{ C}^{-1}$$

$$\approx 0.6 \times 10^6 \text{ N m}^2 \text{ C}^{-1} = 6 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$$

- (iv) (b)  $\phi$

**Explanation:** As flux is the total number of lines passing through the surface, for a given charge, it is always the charge enclosed  $\frac{Q}{\epsilon_0}$ . If area is doubled, the flux remains the same.

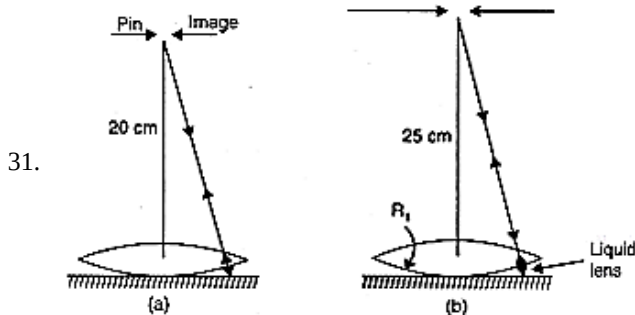
OR

- (c) zero

**Explanation:** As net charge on a dipole is  $(-q + q) = 0$

Thus, when a gaussian surface encloses a dipole, as per Gauss's theorem, electric flux through the surface.

Section E



31.

In the first case, the image will coincide with the pin if the rays from the pin, after refraction through the lens, fall normally on the mirror and retrace their path, as shown in fig (a). This means that the focal length of the convex lens is 20 cm, i.e.  $f_1 = 20 \text{ cm}$

In the second case, the focal length  $F$  of the combination of the convex lens and the plane liquid lens is 25 cm [see fig (b)] i.e.  $F = 25 \text{ cm}$

Let  $f_2$  be the focal length of the liquid lens, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1} = \frac{1}{25} - \frac{1}{20}$$

$$\text{or } f_2 = -100 \text{ cm}$$

For the liquid lens,  $R_1 = -33 \text{ cm}$ , the radius of curvature of the common surface and  $R_2 = \infty$ , if  $\mu$  is refractive index of the liquid.

$$\frac{1}{f_2} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

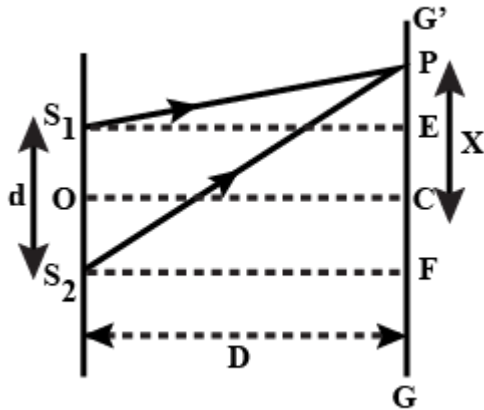
$$\text{or } -\frac{1}{100} = (\mu - 1) \left( -\frac{1}{33} - \frac{1}{\infty} \right)$$

$$\text{or } \mu - 1 = \frac{33}{100} = 0.33$$

$$\text{or } \mu = 1.33$$

OR

i. In Young's double slit experiment. The wavefronts from the two illuminated slit superpose on the screen. When light waves from two illuminated slits is incident on the screen, the path traveled by each light wave is different. This path difference leads to a phase difference in the two light waves. The path difference is different for each point on the screen and hence, intensity is different for all the points. This leads to the formation of bright and dark fringes on the screen



Consider point P on the screen as shown in the figure.

$$S_2P^2 = S_2F^2 + PF^2$$

$$S_2P = \sqrt{D^2 + \left(x + \frac{d}{2}\right)^2}$$

Similarly,

$$S_1P = \sqrt{D^2 + \left(x - \frac{d}{2}\right)^2}$$

Path difference is given by:

$$S_2P - S_1P = \sqrt{D^2 + \left(x + \frac{d}{2}\right)^2} - \sqrt{D^2 + \left(x - \frac{d}{2}\right)^2}$$

Using binomial expansion,

$$S_2P - S_1P = D \left( 1 + \frac{1}{2} \left( \frac{x}{D} + \frac{d}{2D} \right)^2 + \dots \right) - D \left( 1 + \frac{1}{2} \left( \frac{x}{D} - \frac{d}{2D} \right)^2 + \dots \right)$$

Ignoring higher-order terms,

$$\Delta x = S_2P - S_1P \approx \frac{xd}{D}$$

For constructive interference i.e. bright fringes,

$$n\lambda = \frac{xd}{D}$$

$$x_n = \frac{n\lambda D}{d} \quad \text{thus for the dark fringes } \frac{xd}{D} = \frac{(2n-1)\lambda}{2} \quad \text{this implies } x_n = \left(n - \frac{1}{2}\right) \frac{D\lambda}{d}$$

Fringe width is equal to the distance between two consecutive maxima.

$$\beta = x_n - x_{n-1} = \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d}$$

$$\beta = \frac{\lambda D}{d}$$

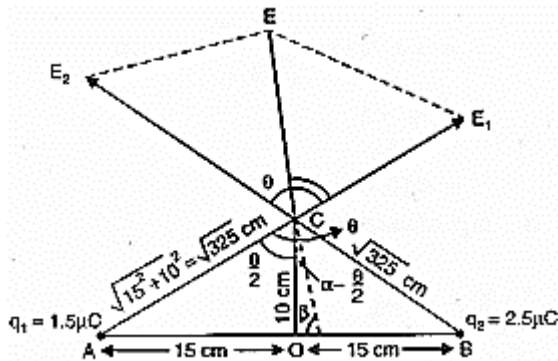
$$\text{ii. We have } \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{25}{9}$$

$$\therefore \frac{a_1 + a_2}{a_1 - a_2} = \frac{5}{3} \Rightarrow \frac{a_1}{a_2} = \frac{4}{1}$$

$$\therefore \frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{(a_1)^2}{(a_2)^2} = \frac{16}{1}$$

32. a. Potential at the mid point of the line joining the two charges is

$$\begin{aligned}
 V &= \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1}{r_1} + \frac{q_2}{r_2} \right] \\
 &= 9 \times 10^9 \left[ \frac{15 \times 10^{-6}}{0.15} + \frac{2.5 \times 10^{-6}}{0.15} \right] V \\
 &= 9 \times 10^9 \times 10^{-6} \left[ 10 + \frac{50}{3} \right] \\
 &= 9 \times 10^3 \times \frac{80}{3} \\
 \text{Or } V &= 2.4 \times 10^5 V
 \end{aligned}$$



Electric field at the mid point O due to charge at A

$$\begin{aligned}
 &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^2} = 9 \times 10^9 \times \frac{1.5 \times 10^{-6}}{(0.15)^2} \\
 &= 6 \times 10^5 \text{ Vm}^{-1} \text{ along OB}
 \end{aligned}$$

Electric field at the mid point O due to charge at B

$$\begin{aligned}
 &= \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^2} = 9 \times 10^9 \times \frac{2.5 \times 10^{-6}}{(0.15)^2} \\
 &= 10 \times 10^5 \text{ Vm}^{-1} \text{ along OA}
 \end{aligned}$$

Thus, the total electric field at the mid point O is

$$\begin{aligned}
 E &= 10 \times 10^5 - 6 \times 10^5 \\
 &= 4 \times 10^5 \text{ Vm}^{-1} \text{ (along BA)}
 \end{aligned}$$

b. Potential at the point C due to the two charges is

$$\begin{aligned}
 V &= \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1}{r_1} + \frac{q_2}{r_2} \right] \\
 &= 9 \times 10^9 \left[ \frac{1.5 \times 10^{-6}}{\sqrt{3.25 \times 10^{-2}}} + \frac{2.5 \times 10^{-6}}{\sqrt{325 \times 10^{-2}}} \right] V \\
 &= \frac{9 \times 10^9 \times 10^{-6}}{10^{-2}} \cdot \frac{4.0}{\sqrt{325}} V \\
 &= \frac{9 \times 4}{18.02} \times 10^5 V = 2 \times 10^5 V
 \end{aligned}$$

Electric field at C due to charge at A

$$\begin{aligned}
 E_1 &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^2} \\
 &= 9 \times 10^9 \times \frac{1.5 \times 10^{-6}}{(\sqrt{325 \times 10^{-2}})^2} \text{ Vm}^{-1} \\
 &= \frac{9 \times 1.5}{325} \times 10^7 \text{ Vm}^{-1} \\
 &= 4.15 \times 10^5 \text{ Vm}^{-1}
 \end{aligned}$$

Electric field at C due to charge at B

$$\begin{aligned}
 E_2 &= \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^2} \\
 &= 9 \times 10^9 \times \frac{2.5 \times 10^{-6}}{325 \times 10^{-4}} = 6.92 \times 10^5 \text{ Vm}^{-1}
 \end{aligned}$$

If the angle between  $E_1$  and  $E_2$  be  $\theta$  then

$$\begin{aligned}
 \tan \frac{\theta}{2} &= \frac{0.15}{0.10} = 1.5 \\
 \frac{\theta}{2} &= 56.3^\circ \Rightarrow \theta = 112.6^\circ
 \end{aligned}$$

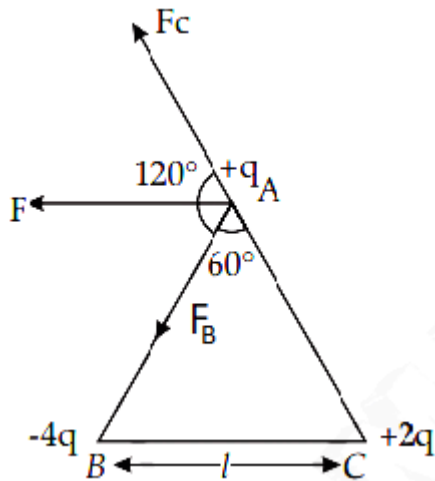
Thus, magnitude of resultant field at C is

$$\begin{aligned}
 E &= \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos \theta} \\
 &= \sqrt{(4.15 \times 10^5)^2 + (6.92 \times 10^5)^2 + 2 \times 4.15 \times 10^5 \times 6.92 \times 10^5 \cos 112.6^\circ} \\
 &= 10^5 \sqrt{17.2 + 47.8 - 2 \times 4.15 \times 6.92 \cos 67.4^\circ} \\
 (\because \cos(180 - \theta) &= -\cos \theta) \\
 &= 10^5 \sqrt{43} = 6.56 \times 10^5 \text{ Vm}^{-1}
 \end{aligned}$$



OR

- i. Consider the figure shown below. The forces acting on charge  $q$  at A due to charges  $-4q$  at B and  $2q$  at C are  $F_1$  along AB and  $F_2$  along CA respectively.



$$|\vec{F}_1| = \frac{1}{4\pi\epsilon_0} \frac{(4q)(q)}{l^2} = \frac{1}{4\pi\epsilon_0} \frac{(4q^2)}{l^2} = \frac{1}{\pi\epsilon_0} \frac{q^2}{l^2}$$

$$|\vec{F}_2| = \frac{1}{4\pi\epsilon_0} \frac{(2q)(q)}{l^2} = \frac{1}{2\pi\epsilon_0} \frac{q^2}{l^2}$$

Thus,  $F_1 = 2F_2$

Now angle between  $\vec{F}_1$  and  $\vec{F}_2$  is  $120^\circ$ . Thus magnitude of the resultant force  $F$  is given by,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 120^\circ}$$

$$F = \sqrt{(2F_2)^2 + F_2^2 + 4F_2^2 \cos 120^\circ}$$

$$F = \sqrt{4F_2^2 + F_2^2 - 2F_2^2}$$

$$F = \sqrt{3F_2^2}$$

$$F = \frac{\sqrt{3}}{2\pi\epsilon_0} \frac{q^2}{l^2}$$

- ii. The amount of work done to separate the charges to infinity will be equal to potential energy of the system of charges.

$$U = \frac{1}{4\pi\epsilon_0 l} [q \times (-4q) + (q \times 2q) + (-4q \times 2q)]$$

$$U = \frac{1}{4\pi\epsilon_0 l} [-4q^2 + 2q^2 - 8q^2]$$

$$U = \frac{1}{4\pi\epsilon_0 l} [-10q^2]$$

$$U = -\frac{1}{4\pi\epsilon_0 l} [10q^2]$$



$$V = V_0 \sin(1000t + \phi) \Rightarrow \omega = 1000 \text{ Hz}$$

$$R = 400 \Omega, C = 2 \mu\text{F}, L = 100 \text{ mH}$$

Capacitive reactance,  $X_C = \frac{1}{\omega C}$

$$\Rightarrow X_C = \frac{1}{1000 \times 2 \times 10^{-6}}$$

$$\Rightarrow X_C = \frac{10^3}{2} \Rightarrow X_C = 500 \Omega$$

Inductive reactance,  $X_L = \omega L$

$$\Rightarrow X_L = 1000 \times 100 \times 10^{-3} \Rightarrow X_L = 100 \Omega$$

So,  $X_C > X_L$

$$\Rightarrow \tan \phi \text{ is negative.}$$

Hence, the voltage lags behind the current by a phase angle  $\phi$ .

Phase difference,  $\tan \phi = -\frac{X_L - X_C}{R}$

$$\tan \phi = \frac{100 - 500}{400} \Rightarrow \tan \phi = -\frac{400}{400}, \tan \phi = -1$$

$$\Rightarrow \tan \phi = -\tan\left(\frac{\pi}{4}\right) \Rightarrow \phi = -\frac{\pi}{4}$$

This is the required value of the phase difference between the current and the voltage in the given series L-C-R circuit.

ii. Suppose, new capacitance of the circuit is  $C'$ . Thus, to have power factor unity

$$\cos \phi' = 1 = \frac{R}{\sqrt{R^2 + (X_L - X_C')^2}}$$

$$\Rightarrow R^2 = R^2 + (X_L - X_C')^2$$

$$\Rightarrow X_L = X_C' = \frac{1}{\omega C'} \text{ or } \omega L = \frac{1}{\omega C'}$$

$$\Rightarrow \omega^2 = \frac{1}{LC'} \text{ or } (1000)^2 = \frac{1}{LC'} \quad (\because \omega = 1000)$$

$$\Rightarrow C' = \frac{1}{L \times 10^6} = \frac{1}{100 \times 10^{-3} \times 10^6}$$

$$= \frac{10}{10^6} = \frac{1}{10^5} = 10^{-5}$$

$$\Rightarrow C' = 10^{-5} \text{ F} = 10 \times 10^{-6} \text{ F} = 10 \mu\text{F}$$

As,  $C' > C$ , hence, we have to add an additional capacitor of capacitance  $= 10 \mu\text{F} - 2 \mu\text{F} = 8 \mu\text{F}$  in parallel with previous capacitor.

OR

i. Let at any instant, the current and voltage in an L-C-R series AC circuit is given by

$$V = V_m \sin \omega t \text{ and}$$

$$I = I_m \sin(\omega t + \phi)$$

where  $V_m$  and  $I_m$  are the peak values of the ac voltage and ac current respectively.

The instantaneous power is given by

$$P = VI = I_m \sin(\omega t + \phi) V_m \sin \omega t$$

$$\Rightarrow P = \frac{V_m I_m}{2} [2 \sin \omega t \sin(\omega t + \phi)]$$

$$\therefore P = VI = \frac{V_m I_m}{2} [\cos \phi - \cos(2\omega t + \phi)] \quad \dots (i)$$

$$[\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B)]$$

Work done for a very small time interval  $dt$  is given by

$$dW = P dt$$

$$\Rightarrow dW = V I dt$$

$\therefore$  Total work done over a complete cycle i.e. from 0 to  $T$  is given by,

$$W = \int_0^T V I dt$$

$$\text{But } P_{av} = \frac{W}{T} = \frac{\int_0^T V I dt}{T}$$

$$\Rightarrow P_{av} = \frac{1}{T} \int_0^T V I dt$$

$$= \frac{1}{T} \int_0^T \frac{V_m I_m}{2} [\cos \phi - \cos(2\omega t + \phi)] dt$$

$$= \frac{V_m I_m}{2T} \left[ \int_0^T \cos \phi dt - \int_0^T \cos(2\omega t + \phi) dt \right]$$

$$= \frac{V_m I_m}{2T} [\cos \phi(t)]_0^T - 0 \quad (\text{By trigonometry})$$

$$= \frac{V_m I_m}{2T} \cos \phi \times T = \frac{V_m I_m}{2} \cos \phi$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos \phi$$

$$\Rightarrow P_{av} = V_{rms} I_{rms} \cos \phi$$

This is the required expression.

ii. Power factor,  $\cos \phi = \frac{R}{Z}$

where,  $R$  = resistance and  $Z$  = impedance of the circuit.

Low power factor ( $\cos \phi$ ) implies lower ohmic resistance which implies larger power loss in power system (transmission line), because in power system power,  $P \propto \frac{1}{R}$ .